Digital modulation and mobile radio (V)

3.2 Minimum shift keying and Gaussian minimum shift keying

3.2.1 Mathematical derivation of MSK

When it formulated the GSM standard, the standards committee chose a different approach and stipulated angle modulation with a constant envelope of the RF signal for digital networks. This means that the length of the vector representing the RF signal does not change as it turns through an angle of $\Delta\varphi$ from $\varphi(t_1)$ to $\varphi(t_2)$ over a time interval equal to the bit duration. The end of the vector must, therefore, describe the arc of a circle with an angular speed of $\frac{d\varphi}{dt} = \frac{\Delta\varphi}{T_{bit}}$. After turning through this angle, during which a symbol is transmitted, the vector does not remain stationary in the position it has reached but continues to turn in the same direction, so repeating the signal that has been transmitted, or turns in the opposite direction, so sending the opposite of the transmitted bit.

The phase change of the RF signal with respect to the arbitrary zero phase of the unmodulated carrier is associated with a frequency change of Δf. The instantaneous frequency and the frequency deviation can be calculated from the first derivative of the carrier phase, in other words from the derivative of the argument of the cosine function describing the carrier.

Modulation index h	Frequency deviation Δf	Phase shift over bit duration T_{bit}	Correlation factor ρ	Euclidean distance D
0.5	$\frac{1}{4\,T_{bit}} = \frac{1}{4}\,f_{bit}$	$\frac{\pi}{2} = 90°$	0	$\sqrt{2E_{bit}}$

TABLE 3 MSK parameters

If the phase change $\Delta\varphi$ that occurs over the duration of a bit is expressed as a multiple h of π, h being referred to as the modulation index, the following relations are obtained:

$$\begin{aligned}\Phi(t) &= 2\pi f_c t + \varphi(t)\\ &= 2\pi f_c t + \varphi_0 \pm \frac{2\pi h}{2T_{bit}}\,t,\end{aligned}$$

$$\begin{aligned}\frac{d\Phi(t)}{dt} &= 2\pi f_c + \left(\frac{d\varphi(t)}{dt}\right)\\ &= 2\pi f_c \pm 2\pi\Delta f\\ &= 2\pi f_c \pm \frac{2\pi h}{2T_{bit}};\end{aligned}$$

$$\Delta f = \frac{h}{2T_{bit}};$$

$$\varphi(t) = \varphi_0 \pm \frac{2\pi h}{2T_{bit}}\,t; \quad \Delta\varphi = h\pi \qquad (19)$$

The signals representing the symbols 0 and 1 can then be expressed as:

$$\begin{aligned}s_1(t) &= A\cdot\cos\left(2\pi\left(f_c + \frac{h}{2T_{bit}}\right)t\right)\\ &= A\cdot\cos(2\pi(f_c + \Delta f)t)\end{aligned}$$

and

$$\begin{aligned}s_2(t) &= A\cdot\cos\left(2\pi\left(f_c - \frac{h}{2T_{bit}}\right)t\right)\\ &= A\cdot\cos(2\pi(f_c - \Delta f)t) \qquad (20)\end{aligned}$$

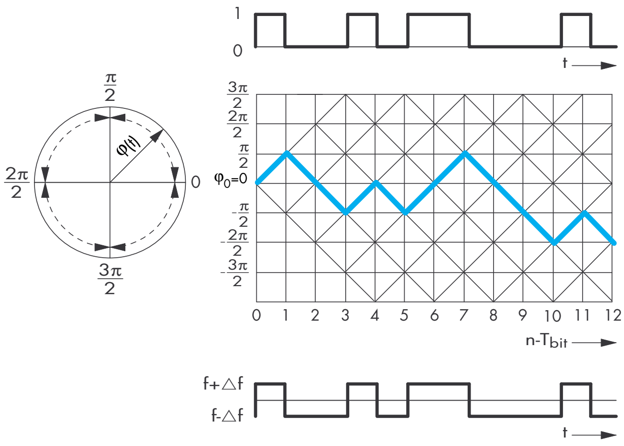

FIG 10 Relationships between data signal, frequency and phase for minimum shift keying

These equations describe frequency keying (see equation 12). Because the phase changes continuously, this type of modulation is referred to as continuous phase frequency-shift keying (CPFSK). FIG 10 shows the correlations between the data sequence, current phase and current frequency. The trellis diagram which shows the possible phase transitions also explains why a CPFSK modulator has a memory. It is easy to see how the current phase depends on previous states.

Equation 13 gives the following expression for the correlation factor of the two signals $s_1(t)$ and $s_2(t)$:

$$\rho = \frac{\sin(\pi\cdot h)}{\pi\cdot h} - \frac{\sin(2\pi f_c T_{bit})}{2\pi f_c T_{bit}} \qquad (21)$$

If $f_c \gg 1/T_{bit}$, the second term can be neglected. Uncorrelated signals ($\rho = 0$) are obtained when $h = k\cdot 0.5$ where $k \in \{1, 2, 3, \ldots\}$; $h = 0.5$ is therefore the smallest modulation index for which the two signals are uncorrelated. FSK with $h = 0.5$ and $\Delta f = \frac{1}{4\,T_{bit}} = \frac{1}{4}\,f_{bit}$ is therefore also referred to as minimum shift keying (MSK). TABLE 3 summarizes the main parameters for this type of modulation once more.

3.2.2 Implementing MSK

The simplest way of producing an MSK signal would be to convert the data sequence a(n) into a bipolar NRZ signal which is then used to control a VCO (FIG 11). This approach is in fact adopted for, say, cordless phones where the specifications for the frequency and angle accuracy of the

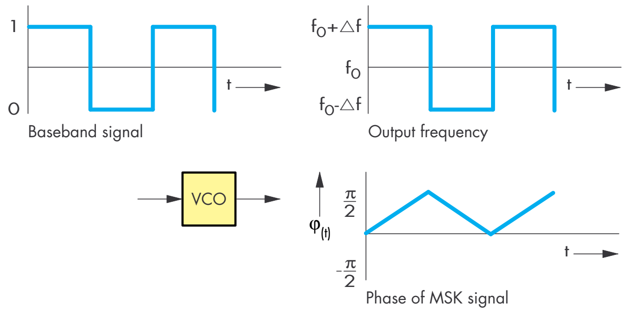

FIG 11 Generating minimum shift keying with VCO

modulated signal do not have to be so stringent. However, the GSM specifications for the maximum frequency and angle error during a burst are so tight that a VCO would be incapable of meeting them. An I/Q modulator gives better results under these restraints. Before analyzing a modulator of this kind, it will be useful to briefly review the I/Q notation used for modulated RF carriers once more.

$$\begin{aligned} s(t) &= A \cdot \cos[2\pi f_c t + \varphi(t)] \\ &= A \cdot [\cos(\varphi(t)) \cdot \cos(2\pi f_c t) \\ &\quad + \sin(\varphi(t)) \cdot (-\sin(2\pi f_c t))] \qquad (22) \end{aligned}$$

The RF signal has two mutually orthogonal components $\cos(2\pi ft)$ and $-\sin(2\pi ft) = \cos(2\pi ft + \pi/2)$ which are multiplied by the functions $\cos\varphi(t)$ and $\sin\varphi(t)$ by means of, say, two double-balanced mixers. The angle $\varphi(t)$ can be obtained from equation 19 and is given below:

$$\varphi(t) = \varphi(nT_{bit}) \pm 0.5\pi/T_{bit} \cdot t^* \qquad (23)$$

with $0 < t^* < T_{bit}$

In practise, the modulated carrier can be obtained in the following way:

1. Obtain the two mutually orthogonal carrier components $I(t) = \cos(2\pi ft)$ and $Q(t) = -\sin(2\pi ft)$.
2. Multiply the I component with $\cos\varphi(t)$ and the Q component with $\sin\varphi(t)$.
3. Add the two components.

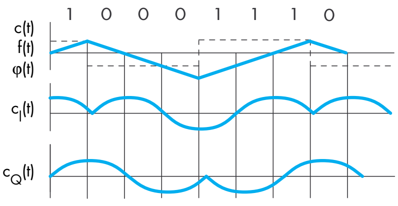

FIG 12 Modulation signals and I/Q signals for MSK

The next problem is the generation of the modulating signals $\cos\varphi(t)$ and $\sin\varphi(t)$. This cannot be done with classic analog methods, the signals have to be calculated digitally using equation 23 for the time $nT_{bit} < t^* < (n+1)T_{bit}$, the polarity of the fraction determining whether a 1 or a 0 is sent.

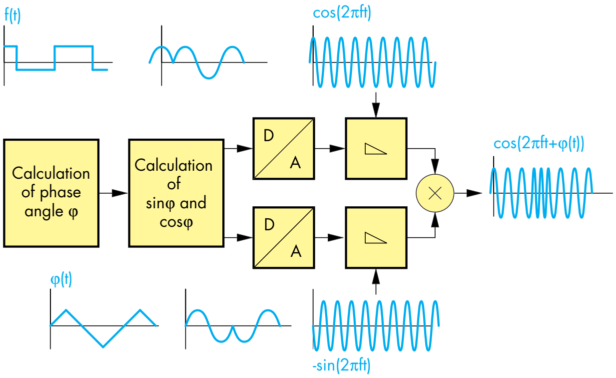

FIG 13 I/Q modulator for MSK

If the data sequence a(n) is mapped onto the data function

$$c(t) = c(nT_{bit} + t^*) = \begin{cases} +1 \text{ for } a(n+1) = \text{“1”} \\ -1 \text{ for } a(n+1) = \text{“0”} \end{cases} \qquad (24)$$

(23) can be rearranged to give

$$\varphi(t) = \varphi(nT_{bit}) + c(t) \cdot \frac{0.5 \cdot \pi}{T_{bit}} t^* \qquad (25)$$

This yields:

$$c_I(t) = \cos(\varphi(nT_{bit})) \cdot \cos\left(\frac{\pi}{2\,T_{bit}} t^*\right)$$

$$- c(t) \cdot \sin(\varphi(nT_{bit})) \cdot \sin\left(\frac{\pi}{2\,T_{bit}} t^*\right)$$

and

$$c_Q(t) = \sin(\varphi(nT_{bit})) \cdot \cos\left(\frac{\pi}{2\,T_{bit}} t^*\right)$$

$$+ c(t) \cdot \cos(\varphi(nT_{bit})) \cdot \sin\left(\frac{\pi}{2\,T_{bit}} t^*\right) \qquad (26)$$

The following approach is used:

1. Start with an initial phase. If $\varphi(0) = 0$ is selected, equation 26 gets less complex: $\sin(\varphi(nT_{bit}))$ becomes 0, $\cos(\varphi(nT_{bit}))$ becomes ±1.
2. Calculate $\varphi(t)$ (for example by using a digital accumulator).
3. Calculate the functions of time $c_I(t)$ and $c_Q(t)$ from tables.
4. Calculate the phase $\varphi(nT_{bit})$, (last accumulator value in 2).
5. Go to 2.

FIG 12 shows this procedure for eight consecutive bits; FIG 13 shows how the modulator operates. The bipolar NRZ signal is proportional to the instantaneous output frequency $f_c \pm \Delta f$ of the modulator. In the accumulator it is integrated to give a signal that is proportional to the instantaneous phase of the modulated carrier. Tables for $\sin\varphi(t)$ and $\cos\varphi(t)$ provide the signals $c_I(t)$ and $c_Q(t)$ in digital form. After D/A conversion and analog filtering by a lowpass, these signals are fed to the modulators at whose RF inputs the orthogonal components of the carrier have been applied. The two modulated carrier components are added in a power summer to give the output signal. If you consider the I/Q modulator from the point where the bipolar NRZ signal is fed into the RF output, it acts as a frequency modulator or a VCO.

To be continued. Peter Hatzold