Digital modulation and mobile radio (VI)

3.2.3 Bandwidth reduction by baseband filtering

The power-density spectrum of unfiltered MSK can be described analytically by the function

$$\Phi_{vv(MSK)} = \frac{16\,A^2\,T_{bit}}{\pi^2}\left[\frac{\cos 2\,\pi\,f\,T_{bit}}{1-16\,f^2\,T_{bit}^2}\right]^2 \quad (27)$$

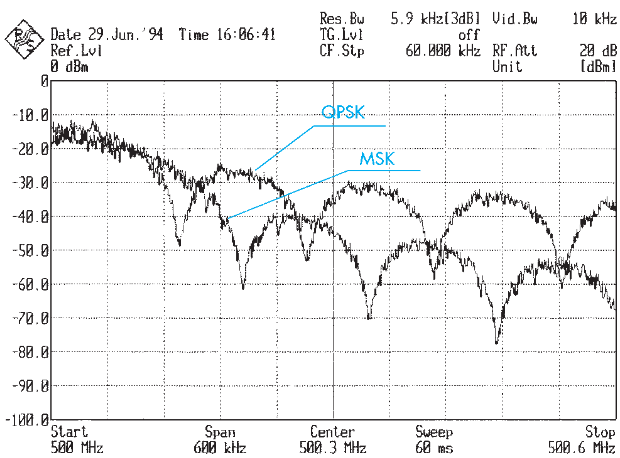

FIG 14 Spectra for QPSK and MSK

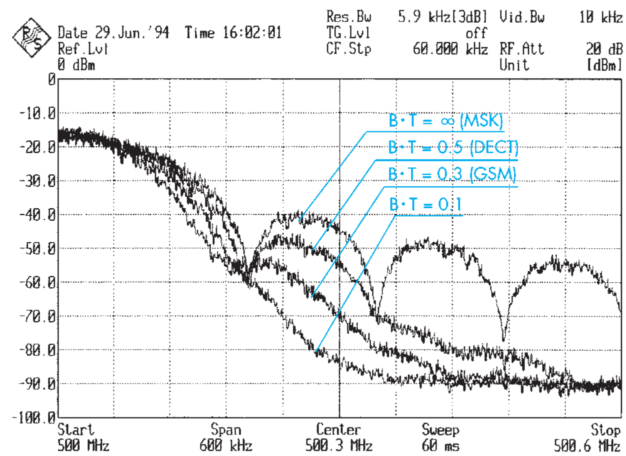

FIG 16 GMSK spectra for various values of bandwidth · bit duration

In FIG 14 it is compared with the QPSK power-density spectrum function. The diagrams show that the main lobe of the MSK spectrum is considerably wider and that there are no spectrum zeroes at f_c +/– f_{bit} . On the other hand, the MSK spectrum's tail-off, which is proportional to f^{-4}, is considerably steeper than that of the QPSK spectrum. In both cases the spectrum can be improved by baseband filtering, but with one big difference – in the case of QPSK it is the modulating signals $c_I(t)$ and $c_Q(t)$ that are filtered, but with MSK the data function is filtered before the modulating signals $c_I(t)$ and $c_Q(t)$ are calculated from it by means of a nonlinear operation.

For the time being it will be convenient to think of the I/Q modulator as a frequency modulator (VCO) as far as the processing of the modulating signals is concerned, because this simplifies the description of filtering. Simply imagine the filter connected to the input of the frequency modulator (FIG 15). GSM specifications stipulate that the data signal should be passed through a Gaussian filter, hence the designation Gaussian minimum-shift keying (GMSK) for this type of bandlimited modulation. This filter can be described in terms of its impulse response

$$h(t) = \frac{1}{\sigma\,T_{bit}\,\sqrt{2\,\pi}}\; e^{\frac{t^2}{2(\sigma T_{bit})^2}} \quad (28)$$

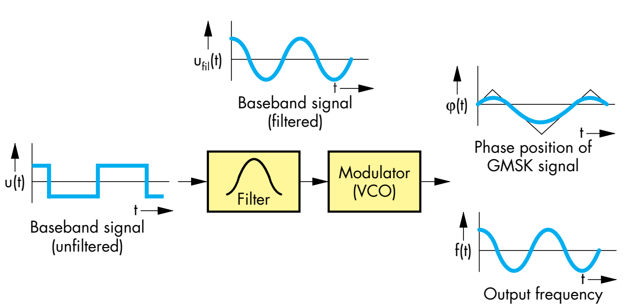

FIG 15 Generation of GMSK

or by its transfer function

$$H(f) = e^{-\frac{\ln 2}{2\,B^2}f^2} = e^{-\frac{\ln 2}{2(B\cdot T_{bit})^2}(T_{bit}\,f)^2} \quad (29)$$

where $\sigma = \frac{\sqrt{\ln 2}}{B\cdot T_{bit}}$ and B = 3 dB bandwidth of filter. These expressions contain the new term $B\cdot T_{bit}$, which normalizes the filter bandwidth to the bit frequency f_{bit} and which is used instead of the actual bandwidth of the Gaussian filter to describe the efficiency of the filtering process. $B\cdot T_{bit} = \infty$ means that MSK is being implemented, while smaller values of $B\cdot T_{bit}$ indicate GMSK with a correspondingly smaller bandwidth. FIG 16 shows the effect on the RF spectrum.

GSM networks use $B\cdot T_{bit}$ = 0.3. This means that the 3 dB bandwidth of the baseband signal is 81.25 kHz (TABLE 4).

Apart from the wanted effect of bandlimiting that is obtained by filtering the data function, there is also an unwanted effect referred to as intersymbol interference. Theoretically, when a rectangular pulse $p_c(t) = \mathrm{rect}(t/T_{bit})$ of duration T_{bit} is filtered, its duration t satisfies the inequality $-\infty < t < +\infty$. To estimate the interference, the ap-

Bit duration T_{bit}	Bit frequency f_{bit}	Bandwidth · bit duration $B \cdot T_{bit}$	3 dB bandwidth
3.69 µs	270.833 kHz	0.3	81.25 kHz

TABLE 4 GMSK parameters for GSM

proximate response of the filter to this pulse can be obtained from convolution with the impulse response of the filter. The convolution of $p_c(t) * h(t)$ gives rise to integrals of the form

$$\int_A^B \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx,$$ which do not have

closed-form solutions but can be calculated from the Gaussian error function erf(x) using the methods of numerical analysis (FIG 17).

In practice, when $B \cdot T_{bit} = 0.3$, only an interval from $t = -3T_{bit}$ to $t = +3T_{bit}$, the duration of 6 bits, needs be considered; outside this time interval the filter response can be assumed to be zero. A delay of at least $3T_{bit}$ must be introduced to prevent causality from being violated. Due to pulse spreading and the conservation of energy, the maximum value of the filtered pulse drops to about 0.7 times the amplitude of the stimulus.

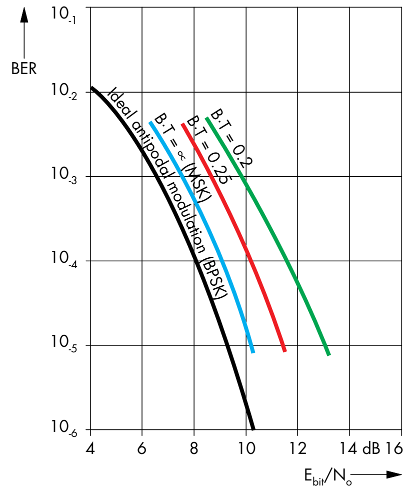

FIG 20 Bit error rate, BER, as function of E_{bit}/N_0 with $B \cdot T$ as parameter

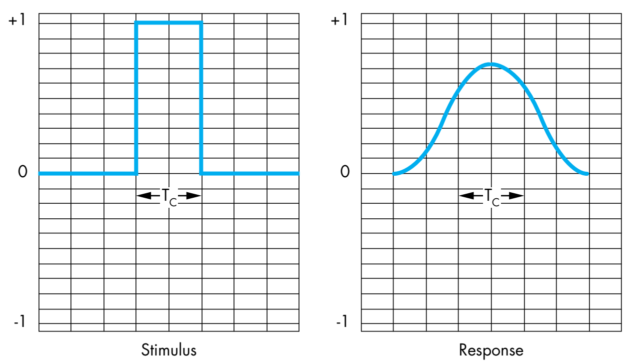

FIG 17 Shaping rectangular pulse with Gaussian filter

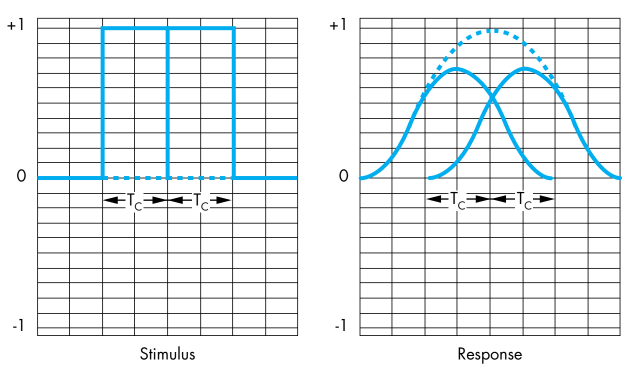

FIG 18 Reinforcement of two neighbouring rectangular pulses with same polarity (resulting output function shown by dashes)

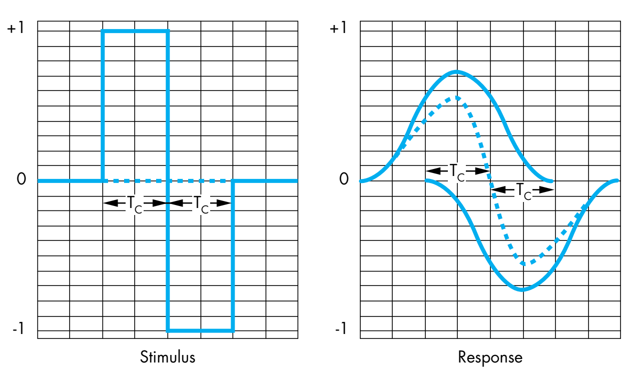

FIG 19 Cancellation of two neighbouring rectangular pulses with opposite polarity (resulting output function shown by dashes)

The responses of the Gaussian filter to neighbouring rectangular pulses reinforce and cancel each other out. Reinforcement occurs if neighbouring pulses have the same polarity (FIG 18) and cancellation, ie the maximum amplitude of the current pulse is reduced even further to about 0.5 times the value of the original pulse, if neighbouring pulses have opposite polarities (FIG 19).

Because of filtering, the function $c_{fil}(t)$, which is proportional to the instantaneous output frequency, is continuous at the modulator input and the phase function $\varphi_{fil}(t)$ loses its breakpoints. This in turn smooths the modulating functions $c_I(t)$ and $c_Q(t)$ and, as a result, there is an improvement in the spectrum that is a function of $B \cdot T$; this is shown in FIG 16.

However, because of intersymbol interference, the improvement in the spectrum has to be traded off against an error rate that increases as $B \cdot T$ decreases, the ratio E_{bit}/N_0 remaining constant (FIG 20).

To be concluded. Peter Hatzold